Check Your Understanding

Limits and Continuity:

1. Construct a function with a jump discontinuity at $x = 2$
2. Construct a function with an infinity discontinuity at $x = 5$

Algebraic Evaluation:

1. Evaluate
$$\lim_{x\to k} \frac{1}{x-k}$$
where k is some positive constant.
   (a) Does your answer change if k is negative?
2. $\lim_{x\to 3} x^2 + 3x + 5$

Limits at Infinity:

1. Evaluate the following limits
   (a) $\lim_{x\to\infty} \dfrac{x^3+4x+5}{x^2+3x+2}$
   (b) $\lim_{x\to\infty} \dfrac{3x^3+2x+1}{2x^3+x+1}$
   (c) $\lim_{x\to\infty} \sin x$
   (d) $\lim_{x\to\infty} \dfrac{5x+1}{x^2+x+2}$

Piecewise Function

1. Find constants a and b such that the following function is continuous for all x:
$$f(x) = \begin{cases} ax^4+bx^3+4 & x \le 1 \\ -ax-bx^2 & 1 < x < 2 \\ ax^3+b & x \ge 2 \end{cases}$$

Power, Addition, Subtraction, and Constant Rules:

1. Find the derivative of the following functions:
   (a) $f(x) = x^3 + 4x^2 + 5$
   (b) $f(x) = x^5 + x^4$
2. Evaluate the derivative of the following functions at $x = 1$
   (a) $f(x) = 1$
   (b) $f(x) = 2x + x^6$

Product and Quotient Rules and Special Derivatives:

1. Find the derivative of the following functions:
   (a) $f(x) = e^x \sin x$
   (b) $f(x) = \dfrac{x^2+2x+1}{\ln x}$

Implicit and Chain Rule:

1. Find dy/dx
   (a) $y^2 + \sin y + 3x^2 = \cos 3x$
   (b) $2x^2y + e^{xy} = 0$

Mean Value Theorem:

1. Does the Mean Value Theorem guarantee $f'(x) = 5$ at some point in the interval $[0, 2]$ for the function $f(x) = x^3 + 3x$?
   (a) Take the derivative of the previous function. Calculate $f'(0)$ and $f'(2)$. Use the Intermediate Value Theorem to show that $f'(x) = 5$ for some point in the interval $[0, 5]$
2. Does Rolle's Theorem guarantee a local extrema on the interval $[0, \pi]$ for the function $f(x) = \sin x$?

Extrema and Inflection Points:

1. Find and classify all extrema for the following functions on the given intervals using the First Derivative Test. Verify your answers using the Second Derivative Test. Then, find all inflection points.
   (a) $f(x) = x^4 - 13x^2 + 36$ on the interval $[-5, 5]$.
   (b) $f(x) = \sec x$ on the interval $[-\pi/2, 5\pi/4]$

Related Rates:

1. Find the rate at which the area of a circle expands when $r = 2$ cm given that $dr/dt = 2\pi$ cm/s
2. A ladder slides down a wall. The ladder is 12 ft long. At some time, the rate at which it slides down is $dy/dt = -3$ ft/s (the negative implying that it is sliding down) and the base of the ladder slides along the ground at a rate $dx/dt = 2$ ft/s. Find the distance x between the base of the ladder and the wall and the distance y between the top of the ladder and the ground at this time.

   Hint: Draw a diagram and identify the knowns.

Riemann Sums:

1. Estimate the area under $f(x)$ on the interval $[0, 8]$ using four equal subintervals. Use the Left, Right, and Trapezoidal Riemann Sums.

x	$f(x)$
0	2
2	5
4	0
6	8
8	10

2. Estimate the area under $f(x) = x^4$ on the interval $[0, 8]$ using four equal subintervals. Use a Midpoint Riemann Sum.

Integration Rules:
Find the following antiderivatives.

1. $\int 4x^3 + 6x^2 - 1\, dx$
2. $\int x^{7/3} + x^{5/2} + x^{-1/2}\, dx$
3. $\int 2 \sin x + 3 \cos x\, dx$

u-substitution:
Use u-substitution to find the following antiderivatives.

1. $\int 2x(x^2 - 9)^3\, dx$
2. $\int \sqrt{3x - 1}\, dx$

   Hint: Use the Power Rule
3. $\int \sec^2 2x \tan 2x\, dx$

   Hint: Use double u-substution.

Area Under and Between Curves:

1. Obtain the area under the curve $f(x) = x^2 + 4x + \cos x$ on the interval $[0, \pi]$
2. Obtain the area between the two curves $f(x) = \ln x$ and $g(x) = \frac{1}{2}x - \frac{1}{2}$

Volumes of Solids of Revolution

1. Obtain the volume of the solid formed by rotating the region bounded by $y = x^3$, $x = 4$, and the x-axis, about the x-axis.
2. Find the volume of the solid formed by rotating the region R between $f(x) = \ln x$ and $g(x) = \frac{1}{2}x - \frac{1}{2}$ about the vertical line $x = -2$.

Differential Equations

1. Find a particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = \frac{\sin x}{y}$ with the initial condition $f(\pi) = 0$

   Hint: Follow the steps in the video.

Motion

1. A particle moves along the x-axis. It's position is given by $v(t) = \cos \pi t$. At time $t = 2$, the particle is at $x = 2$.
   (a) Find the position of the particle at time $t = 0$; $x(0)$
   (b) Find any time(s) t when the particle changes direction on the interval $0 \le t < 2\pi$
   (c) Find the total distance traveled by the particle from $t = 0$ to $t = \pi$
   (d) Write an expression for the average velocity of the particle from $t = 0$ to $t = \pi$

   Hint: Use the Average Value Theorem